The Attack of the Didinium!

A classic example of a predator-prey system is Didinium-Paramecium. Didinium is a ferocious little bugger who likes to consume poor, innocent paramecia – see the picture on the right.

Since these are both organisms that can be easily grown in a test tube, they are particularly attractive for experimentation – indeed, much of the classic work in interspecies dynamics has focused on this system.

An Experiment: Paramecium Alone

The dataset below shows the result of one experiment in which ONLY paramecium were introduced into a test tube. As can be seen, the population initially grows, and then seems to stabilize.
Creating a Model

Imagine that you’ve started to develop a model for this system. You created a stock and flow, and you simplified the resulting difference equation to:

\[ P(t + 1) = P(t) + \Delta P(P(t)) \]

where \( \Delta P(P) \) is the net change in the population over one time step for a given population.

Propose a function...

Working with a neighbor, think about what might be going on in this system physically, and propose a function for \( \Delta P \) that you think would be both sensible physically and that would be consistent with the data. Be prepared to explain and defend your choice of function, and to describe what the different parameters in your model are.

Determine parameters...

Presumably the function you chose has some number of parameters. The next thing we’d like you to do is to try to come up with numbers and units for your parameters. You might be tempted to open your computer and try to find out as much as you can about the lifecycle of the paramecium. This would be interesting, and of some use, but for now, let’s just see what we might be able to figure out from the available data. Be prepared to defend your choices.
Let’s try it out!

We’ll walk through a quick implementation of a model or two. You’re free to do the same on your laptop.
Send in the predators...

The person who ran the experiment above also ran an experiment in which the experimental conditions were similar, but he also introduced Didinium into the test tube. It was a blood bath, really. The graph below gives you some sense of the horror:

Now suppose you’ve created what you think is a logical stock and flow for this system:

You’ve mathematically simplified it like this:

\[ P(t + 1) = P(t) + \Delta P(P(t), D(t)) \]

\[ D(t + 1) = D(t) + \Delta D(P(t), D(t)) \]
And you’ve decided that a first cut at modeling this system would be to use a Lotka-Volterra model. Thus, you’ve come up with the following equations for $\Delta P$ and $\Delta D$:

$$\Delta P = \beta (1 - \frac{D}{D_c})P$$

$$\Delta D = -\gamma (1 - \frac{P}{P_c})D$$

**Suggest values for the parameters $\beta$, $\gamma$, $D_c$, and $P_c$.**
Symbiosis

Symbiosis is a close, long-term interaction between species. More specifically, *mutualistic symbiosis* is an interaction that benefits both species.

Clownfish and anemones have this type of relationship. Residence in an anemone is essential to clownfish reproduction. Clownfish hang out in the tentacles of an anemone, which provide protection from predators that can’t tolerate the anemone’s stings (it is believed that clownfish have a layer of mucus that protects them from anemone stings). Clownfish fecal matter feeds the algae on the anemone’s tentacles and which keeps the anemone healthy. The clownfish also feed on small invertebrates, some of which are anemone predators. There are other factors in the relationship, but the bottom line is that *each species has a higher reproductive rate when they are in this symbiotic relationship*.

There are a number of questions we could answer by modeling this system:

1. How long will clownfish survive in the absence of anemones?
2. Is there a minimum clownfish natural birth rate required for sustainable survival of both populations?
3. Why do we think clownfish are cute and anemones are creepy?
Clownfish and Anemones - A Simple Model

A possible stock and flow diagram for this system looks like this:

![Stock and Flow Diagram](image)

The corresponding difference equations look like this:

\[
C(t + 1) = C(t) + b_C(C(t), A(t)) - d_C(C(t))
\]
\[
A(t + 1) = A(t) + b_A(C(t), A(t)) - d_A(A(t))
\]

where \(b_C, d_C, b_A, \) and \(d_A\) are functions that compute the number of clownfish births and deaths and anemone births and deaths per time step.

Four functions or two?

As you refine these difference equations, you might consider combining the birth and death functions, \(b_C\) and \(d_C\), into a single function: net birth rate, \(\Delta C\). Can you think of one reason this might be a good idea, and one reason it might not be?
**Qualitative Clownfish**

To help you think qualitatively about the relationship between these species, consider the questions below, and use the axes provided to sketch out the behavior you expect.

In answering these questions, suppose you have read, as part of your background research, that clownfish will die out if there are no anemones to protect them, but anemones will still survive (albeit less happily) without the presence of clownfish.

- If there are no clownfish, sketch $\Delta C$ as a function of $A$.
- Sketch additional lines on the same graph corresponding to different fixed values of $C$.
- If there are no anemones, sketch $\Delta C$ as a function of $C$.
- Sketch additional lines on the same graph corresponding to different fixed values of $A$. 

![Graph showing $\Delta C$ as a function of $A$ and $C$.](image)
Quantitative Clownfish

- Is the constant function $\Delta(C) = b_0$ consistent with the sketches you produced above (here $c_0$ is a constant)? Why or why not?

- Is the linear function $\Delta(C) = b_0 + b_1C + b_2A$ consistent with the sketches you produced above? If so, are all the terms necessary? If not, why not?

- Is the quadratic function $\Delta(C) = b_0 + b_1C + b_2A + b_3C^2 + b_4AC + b_5A^2$ consistent with the sketches you produced above? If so, are all the terms necessary? If not, why not?
Conditions

Consider the information you (hypothetically) found indicating that clownfish will die out if there are no anemones to protect them. This information probably informed your answers to the previous questions. How did you use this information? Does it put limits on various parameters? Discuss with your neighbor.

Ballpark it!

Based the information you have and the sketches you made, propose units and ranges of values for the parameters in your model.
Qualitative Anemones

- If there are no anemones, sketch $\Delta A$ as a function of $C$?

- As the number of anemones increases, how will $\Delta A$ change? Sketch additional lines on the same graph corresponding to different values of $A$.

- If there are no clownfish, sketch $\Delta A$ as a function of $A$?

- As the number of clownfish increases, how will $\Delta A$ change? Sketch additional lines on the same graph corresponding to different values of $C$.

- Write down a function $\Delta A(A, C)$ that is consistent with the sketches you made above.
**Conditions**

Consider the information you (hypothetically) found indicating that anemones will still survive (albeit less happily) without the presence of clownfish.

Did this information inform your answers to the previous questions? If so, how? If not, would taking it into account change the form of the functions you’ve proposed? Does it put limits on various parameters? Discuss with your neighbor.

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**Ballpark it!**

Based the information you have and the sketches you made, propose units and ranges of values for the parameters in your model.